# Physics-assisted Generative Adversarial Network for X-Ray Tomography

Zhen Guo

Massachusetts Institute of Technology

MIT MECHE

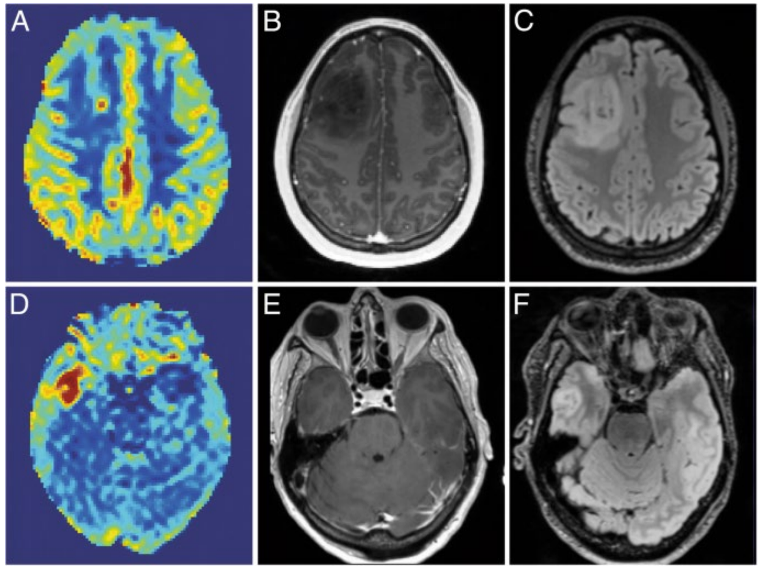
MIT EECS

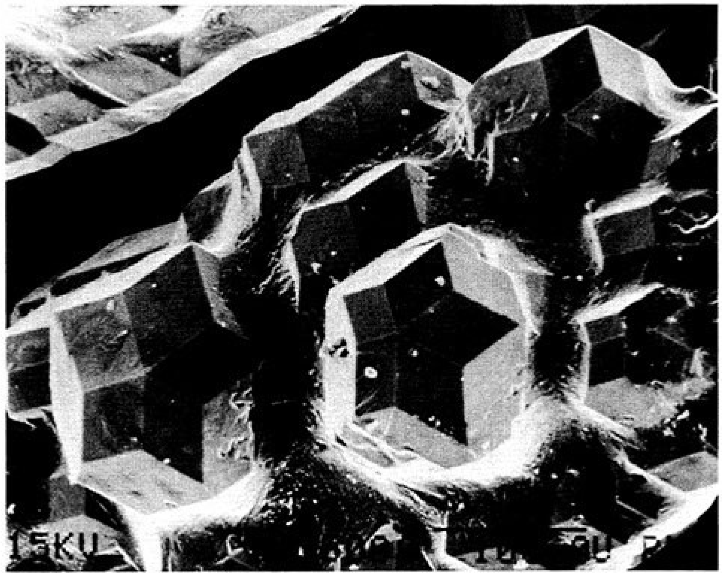
**Zhen Guo, Jung Ki Song, George Barbastathis, Michael E. Glinsky, Courtenay T. Vaughan, Kurt W. Larson, Bradley K. Alpert, Zachary H. Levine**

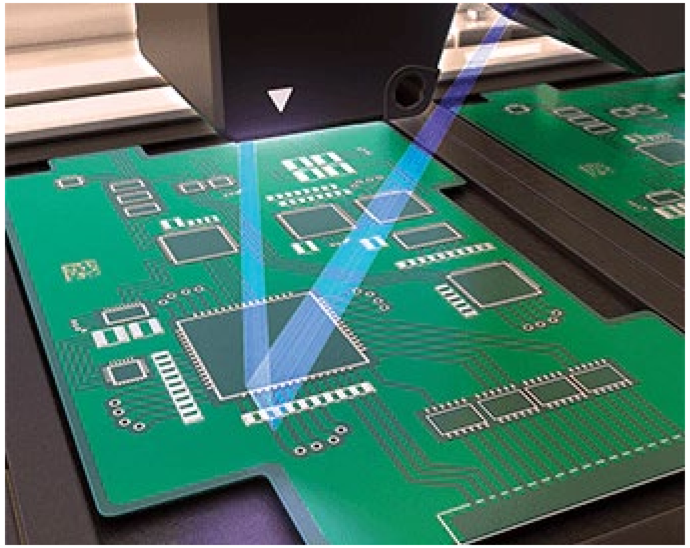
Department of Electrical Engineering and Computer Science, Department of Mechanical Engineering, Massachusetts Institute of Technology,

Singapore-MIT Alliance for Research and Technology (SMART) Centre, Sandia National Laboratory Albuquerque, Applied and Computational Mathematics Division,

National Institute of Standards and Technology, Quantum Measurement Division, National Institute of Standards and Technology





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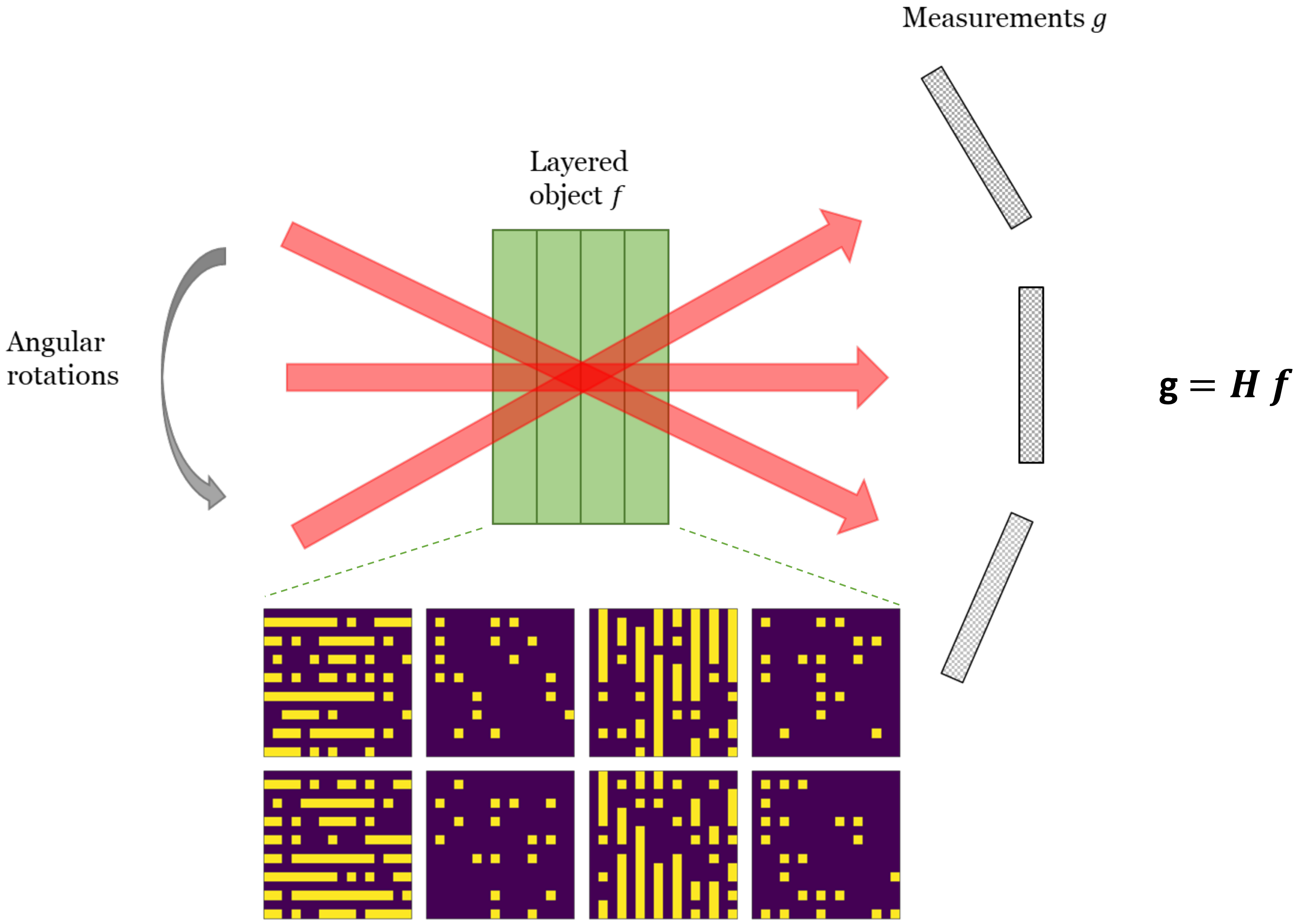


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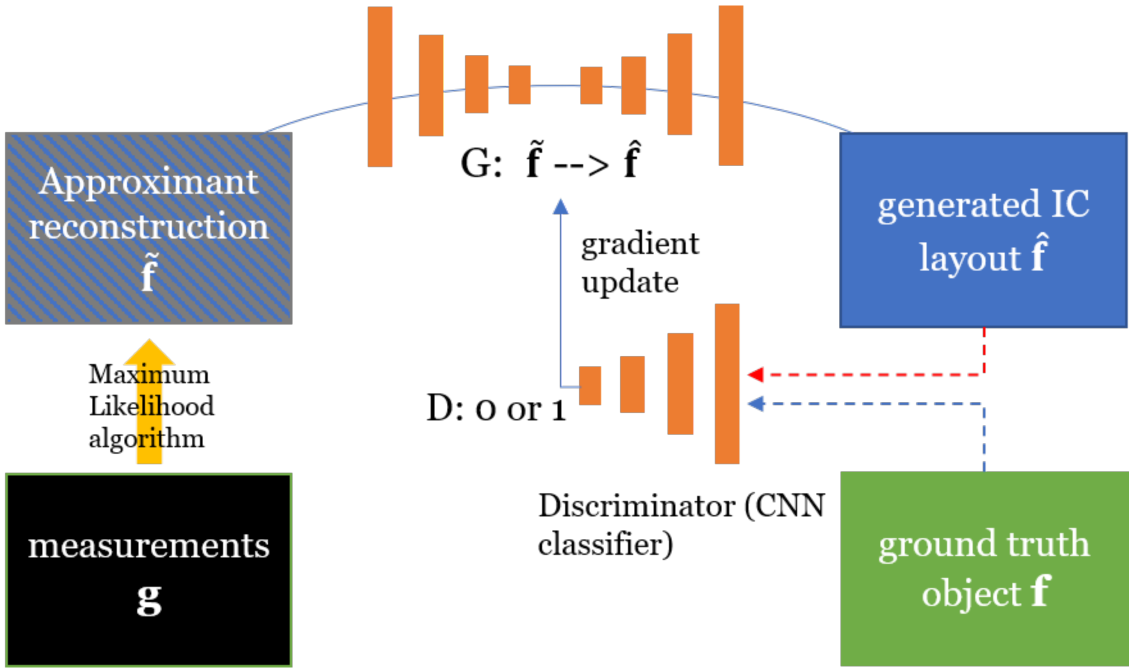


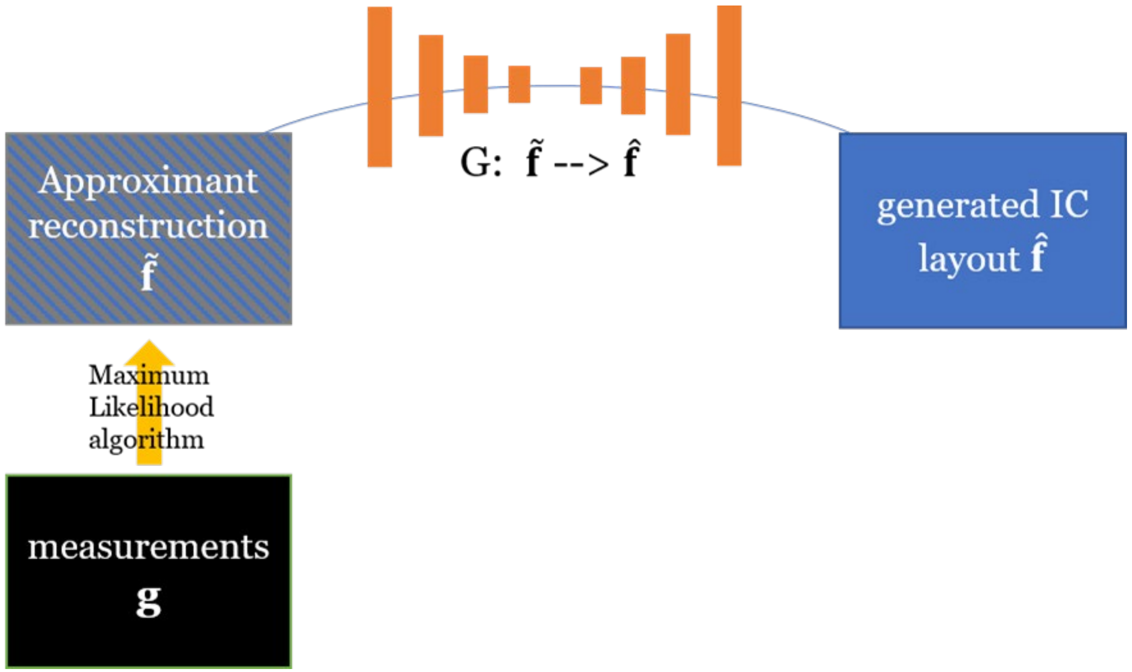


$$\tilde{f}(g) = \arg\max_{f^{(0)}} \left[ L_{\mathrm{MLE}}(g|f^{(0)}) + \Psi(f^{(0)}) \right]$$

$$L_{\mathrm{MLE}}(g|f^{(0)}) = -\sum_i \left[ \ln g_i! - g_i \ln g_i^{(0)} + g_i^{(0)} \right].$$

$$G_{\mathrm{opt}}(\tilde{f}) = \arg\min_G \max_D \mathbb{E}_{(f,\tilde{f})} \left\{ -r_{f,G(\tilde{f})} + \lambda \left[ \log D(f) + \log\left(1 - D(G(\tilde{f}))\right) \right] \right\}$$

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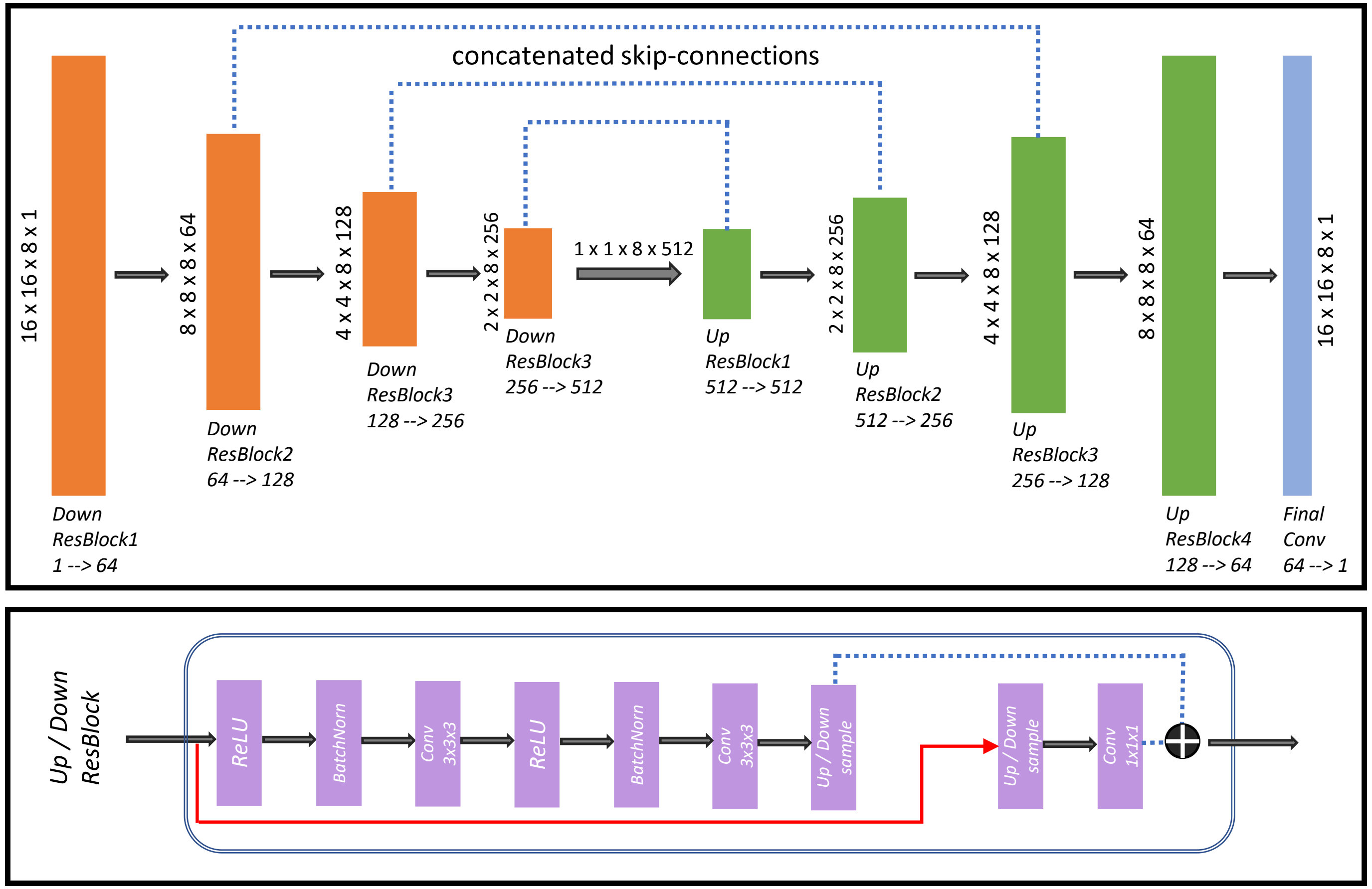


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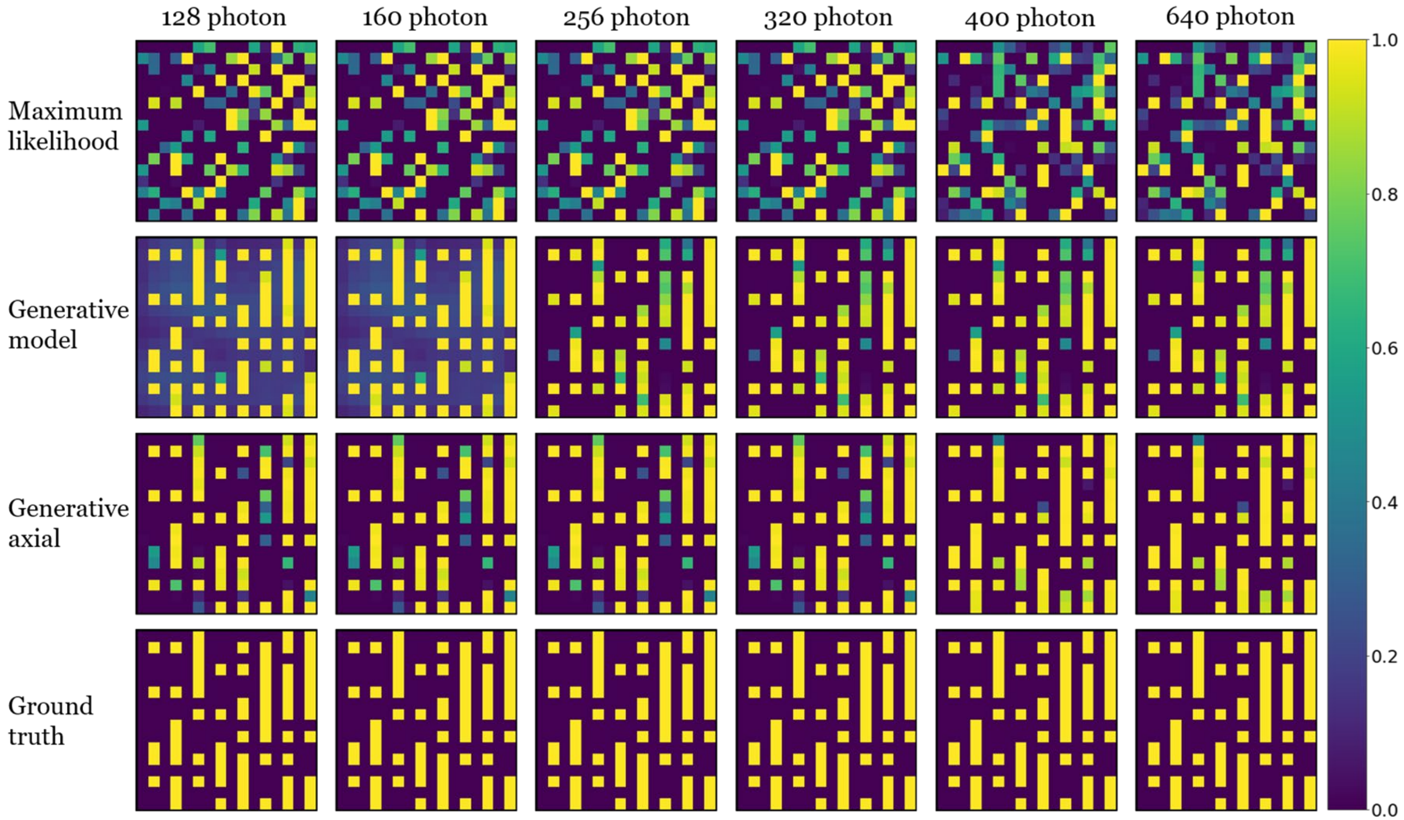


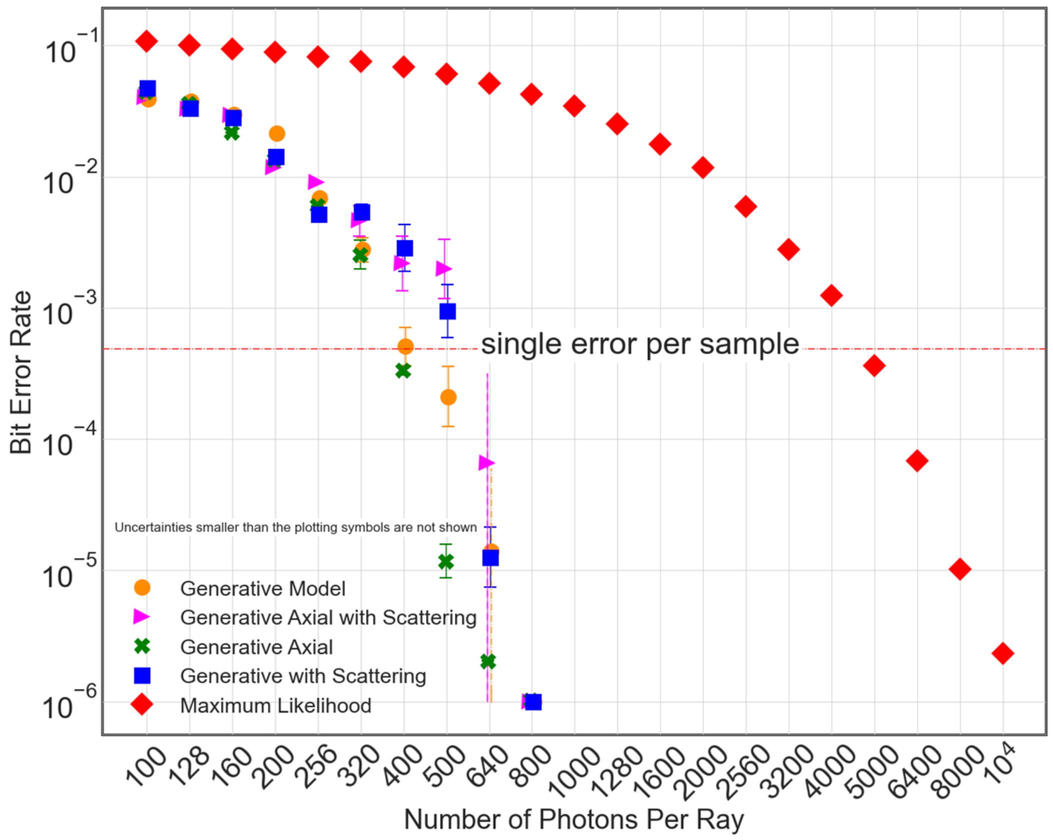


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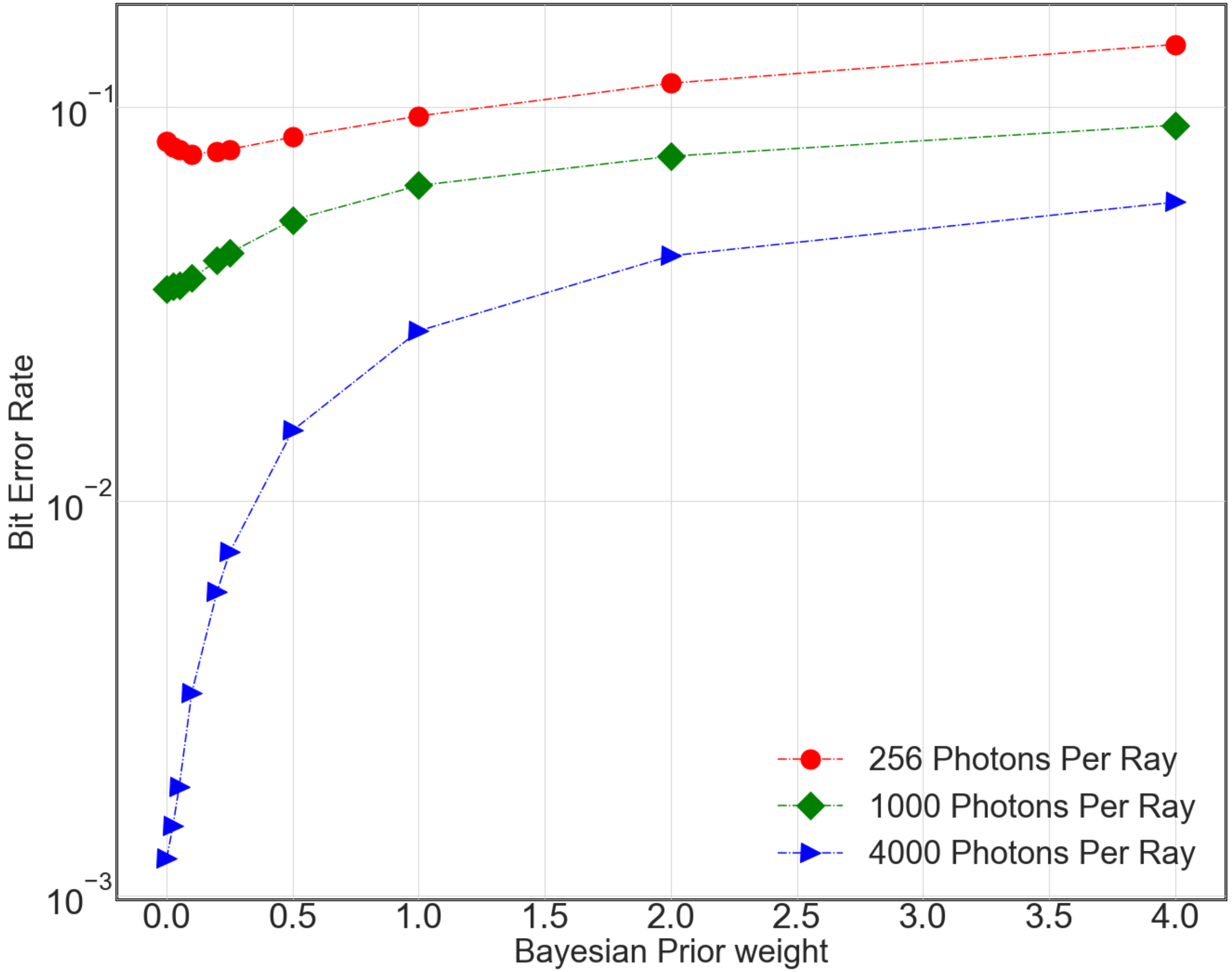


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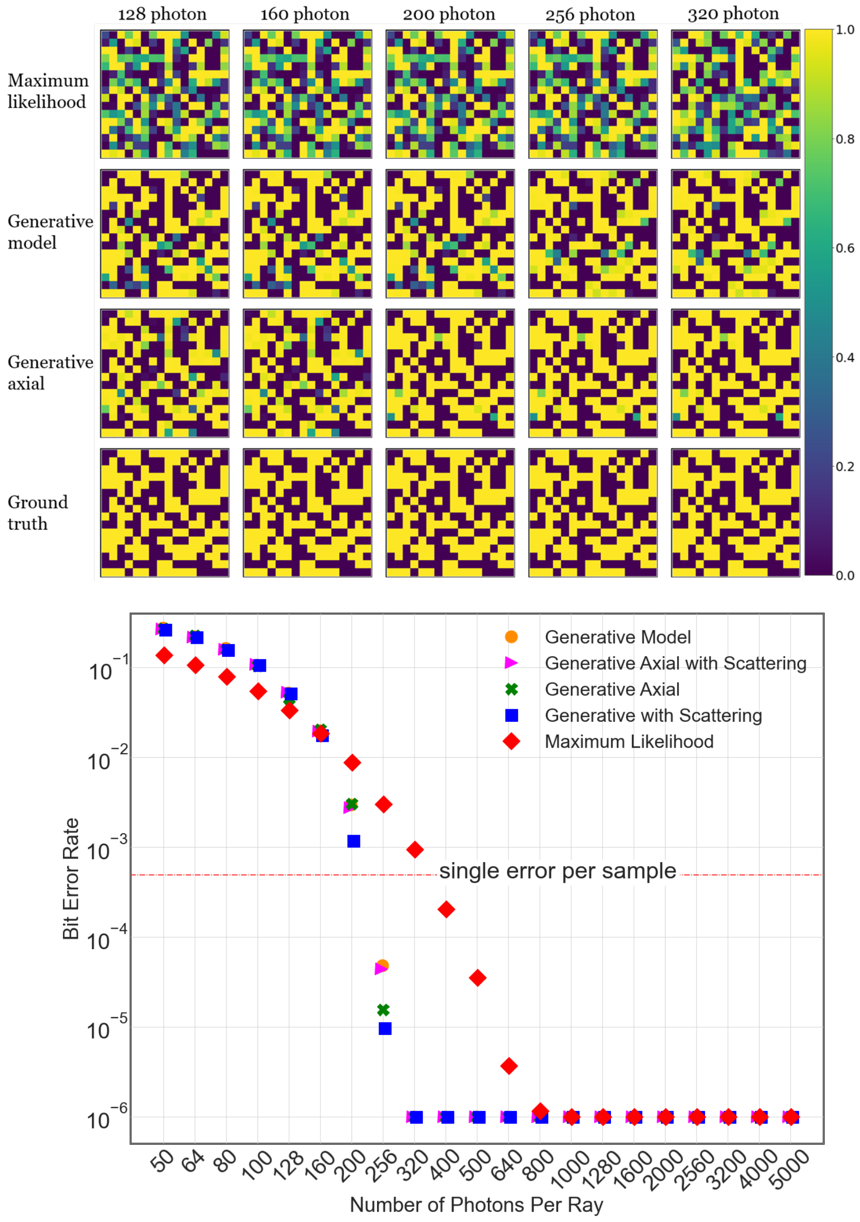
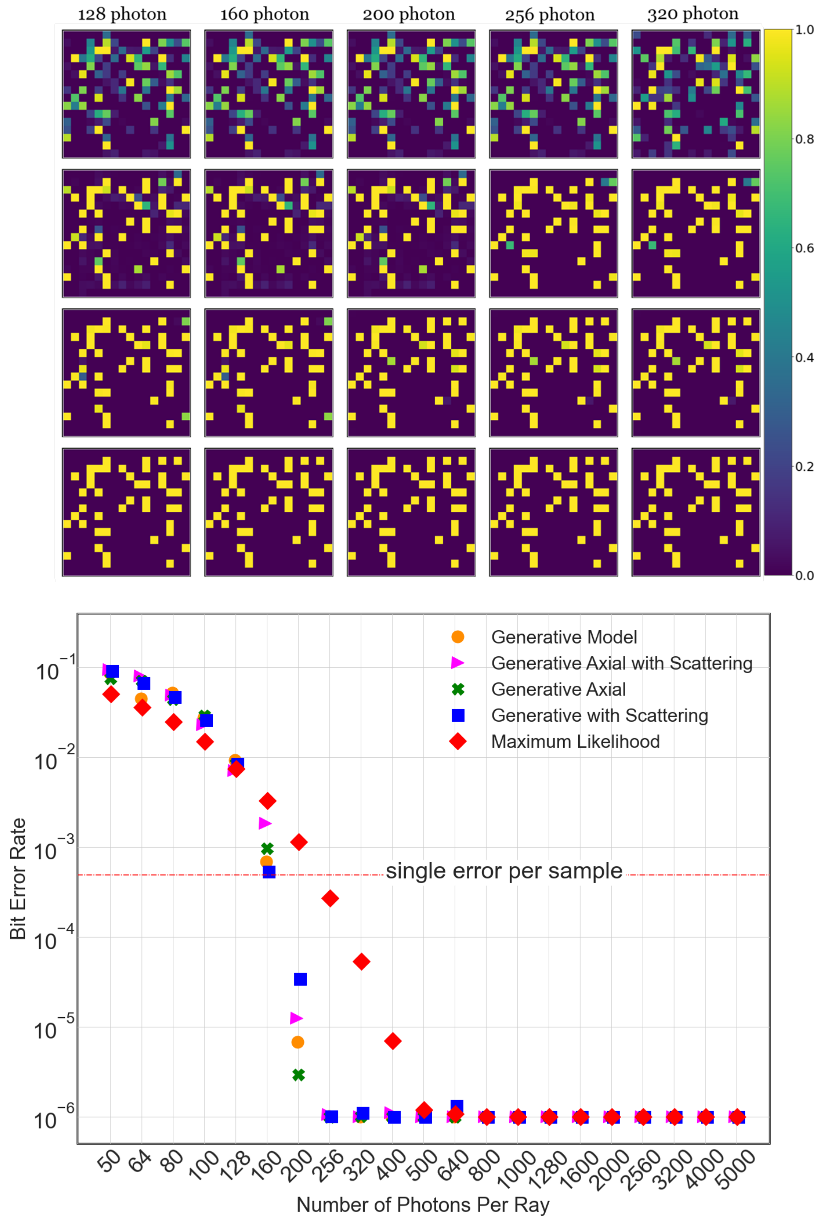
$$\log g(f) = -\lambda^p \left( \sum_i a_i \mid f_i \mid^p + \sum_{\langle ik \rangle} b_{i,k} \mid f_i - f_k \mid^p \right);$$

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# DeepCluster for unsupervised clustering

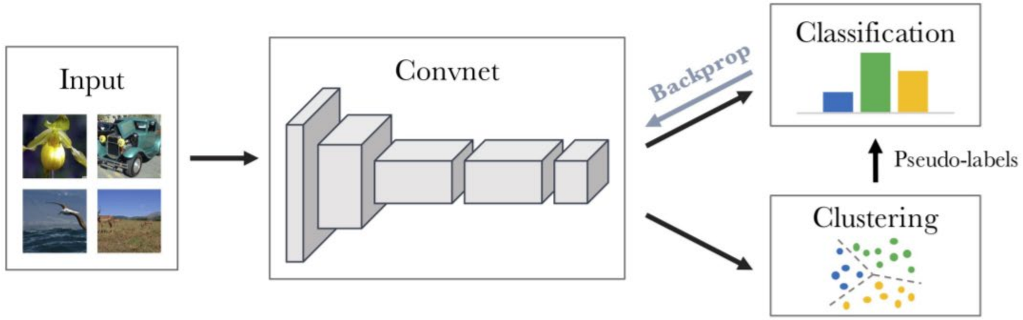
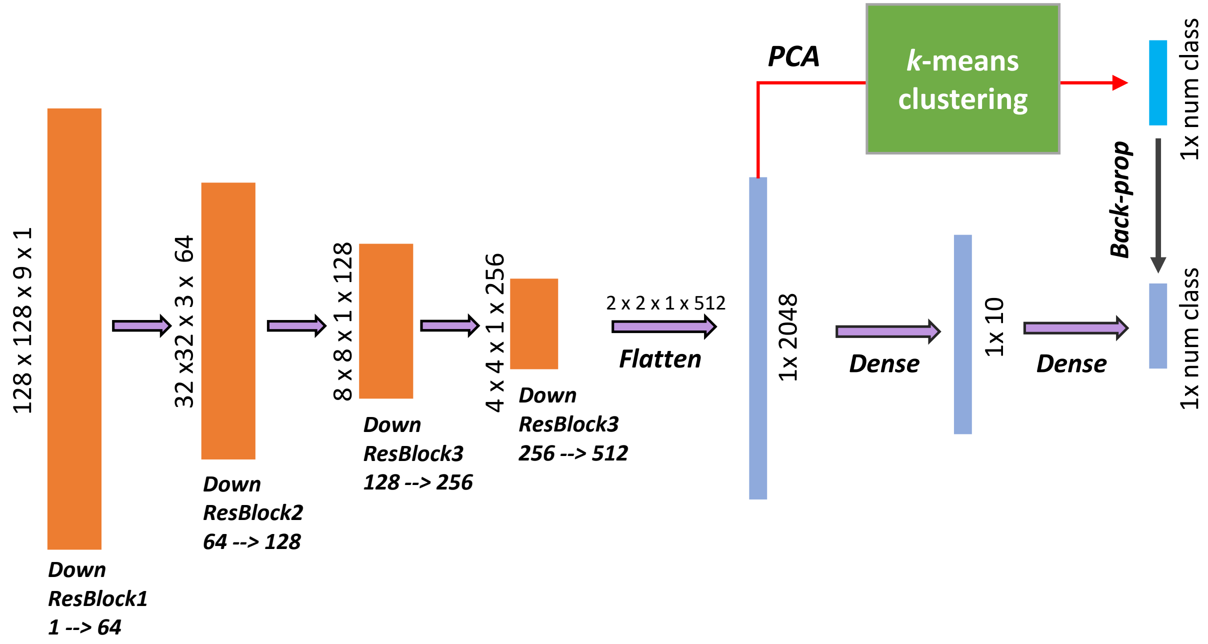



Fig. 1: Illustration of the proposed method: we iteratively cluster deep features and use the cluster assignments as pseudo-labels to learn the parameters of the convnet.

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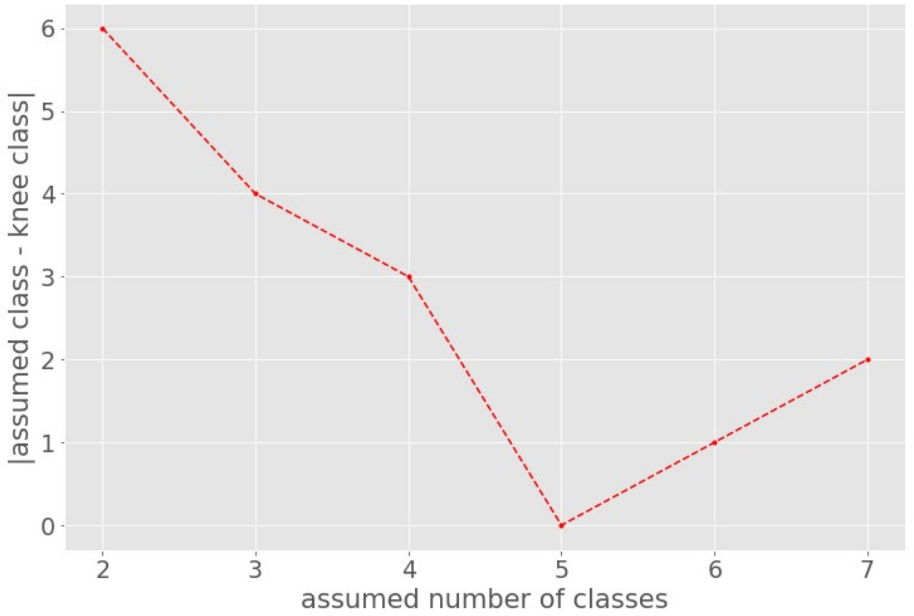
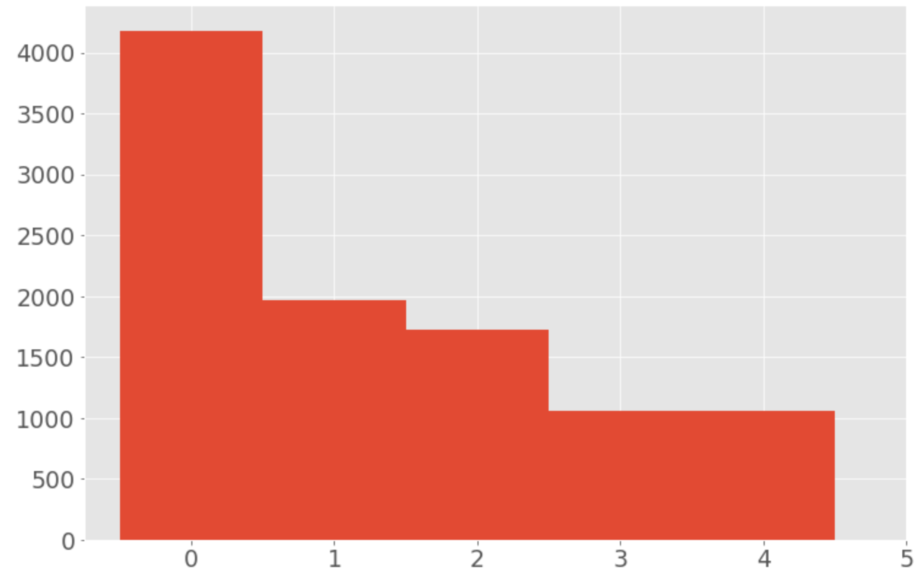
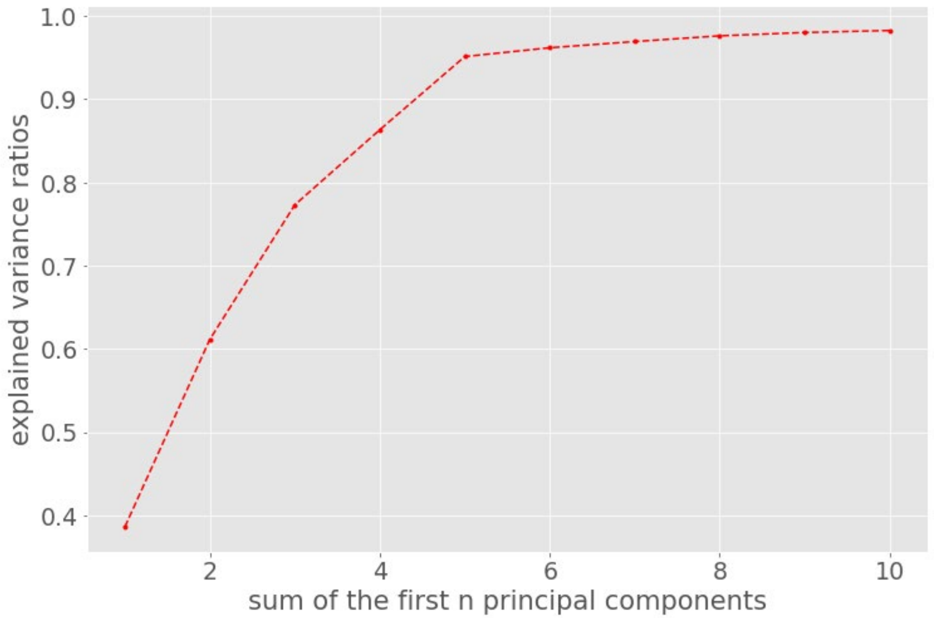
Caron, Mathilde, et al. "Deep clustering for unsupervised learning of visual features." Proceedings of the European conference on computer vision (ECCV). 2018.

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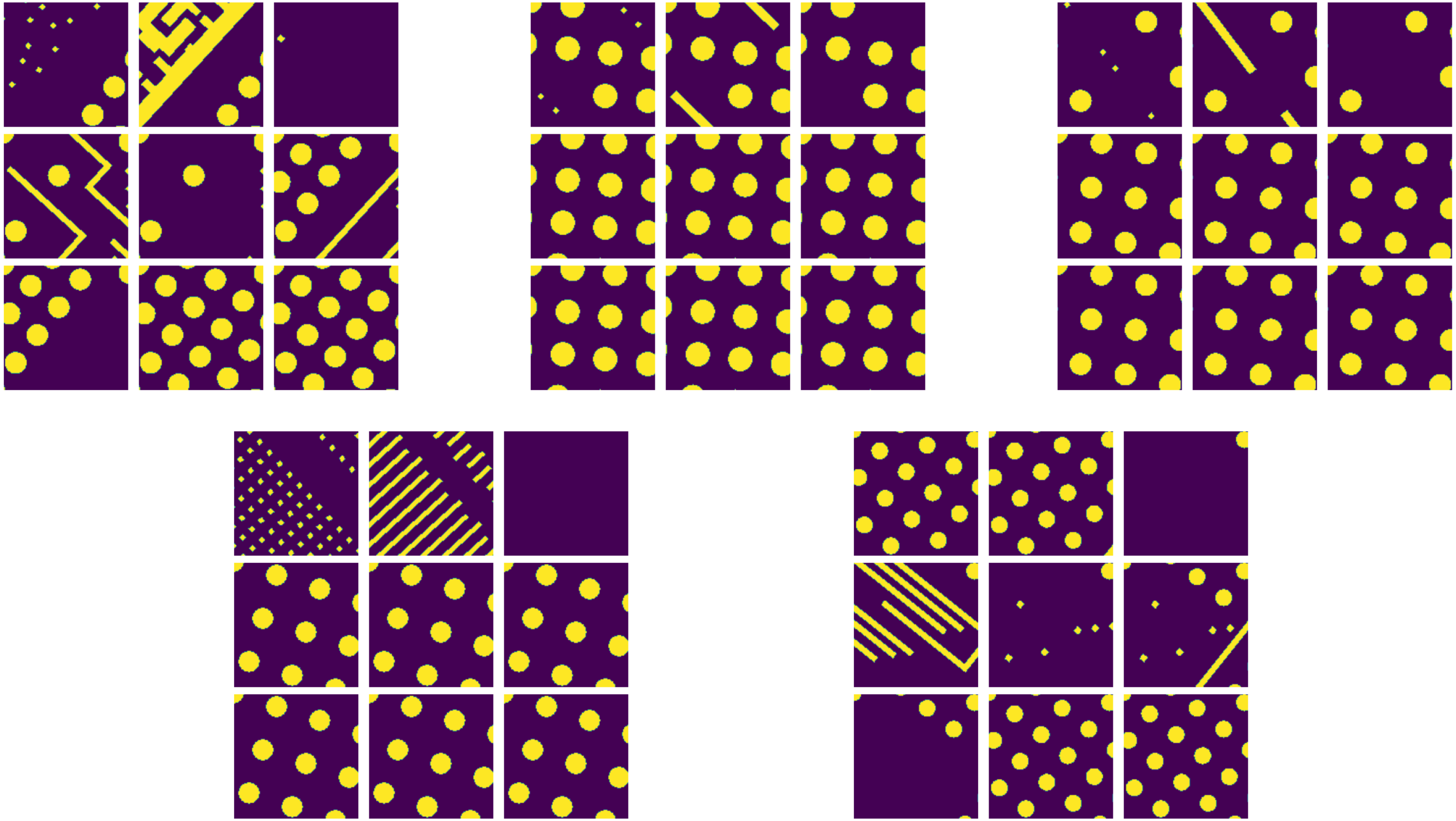

$$p_X(x_j) = \begin{cases} \exp(-\lambda_0)\frac{1}{x_j!}\lambda_0^{x_j} & \text{if } x_j \in R_X \\ 0 & \text{if } x_j \notin R_X \end{cases}$$

The likelihood function is

$$L(\lambda; x_1, \ldots, x_n) = \prod_{j=1}^{n} \exp(-\lambda)\frac{1}{x_j!}\lambda^{x_j}$$

Proof

The $n$ observations are independent. As a consequence, the likelihood function is equal to the product of their probability mass functions:

$$L(\lambda; x_1, \ldots, x_n) = \prod_{j=1}^{n} f_X(x_j; \lambda)$$

Furthermore, the observed values $x_1, \ldots, x_n$ necessarily belong to the support $R_X$. So, we have

$$\begin{aligned} L(\lambda; x_1, \ldots, x_n) &= \prod_{j=1}^{n} f_X(x_j; \lambda) \\ &= \prod_{j=1}^{n} \exp(-\lambda)\frac{1}{x_j!}\lambda^{x_j} \end{aligned}$$

## The log-likelihood function

The log-likelihood function is

$$l(\lambda; x_1, \ldots, x_n) = -n\lambda - \sum_{j=1}^{n} \ln(x_j!) + \ln(\lambda)\sum_{j=1}^{n} x_j$$

Proof

By taking the natural logarithm of the likelihood function derived above, we get the log-likelihood:

$$\begin{aligned} l(\lambda; x_1, \ldots, x_n) &= \ln\left(\prod_{j=1}^{n} \exp(-\lambda)\frac{1}{x_j!}\lambda^{x_j}\right) \\ &= \sum_{j=1}^{n} \ln\left(\exp(-\lambda)\frac{1}{x_j!}\lambda^{x_j}\right) \\ &= \sum_{j=1}^{n} [\ln(\exp(-\lambda)) - \ln(x_j!) + \ln(\lambda^{x_j})] \\ &= \sum_{j=1}^{n} [-\lambda - \ln(x_j!) + x_j \ln(\lambda)] \\ &= -n\lambda - \sum_{j=1}^{n} \ln(x_j!) + \ln(\lambda)\sum_{j=1}^{n} x_j \end{aligned}$$

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